13.1 The Basics of Probability Theory

An **experiment** is a controlled operation that yields a set of results.

The possible results of an experiment are called its **outcomes**. The set of outcomes are the **sample space**.

An **event** is a subcollection of the outcomes of an experiment.

Example:

Rolling a die is an experiment.

The different faces on the die are its outcomes. Sample space = { 1, 2, … 6 }

An event, for example, could be rolling an odd number. E = { 1, 3, 5 }

The probability of an <u>event</u> occurring in an <u>experiment</u> is also a number between (and possibly equal to) 0 and 1.

It is written P(E).

EMPIRICAL ASSIGNMENT OF PROBABILITIES If E is an event and we perform an experiment several times, then we estimate the probability of E as follows:

$$P(E) = \frac{\text{the number of times } E \text{ occurs}}{\text{the number of times the experiment is performed}}.$$

This ratio is sometimes called the *relative frequency* of E.

Example: Experiment is roll a die.

Sample space: { 1, 2, 3, 4, 5, 6 }

What is the probability of rolling an odd number?

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Event E = { 1, 3, 5 }

P(E) = 3/6 = 1/2

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What is the probability of rolling a 5, 6, 7, or 8?

Event E = { 5, 6, 7, 8 }

P(E) = 2/6 = 1/3

Example: Experiment is have 3 children.

Sample space:
{ bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg }

What is the probability of having a girl before a boy?

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Sample space:
{ bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg }

What is the probability of having a girl before a boy?

Event E = { bgb, gbb, bgb, ggb}

P(E) = 4/8 = 1/2

A drug was given with the following results:

Side Effects	Number of Times
None	72
Mild	25
Severe	3

What is the probability of severe side effects?

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$$P(\text{severe side effects}) = \frac{\text{the number of times severe side effects occurred}}{\text{the number of times the experiment was performed}}$$

$$= \frac{3}{100} = 0.03$$

Some probabilities come from running experiments. These are called **empirical probabilities**.

Example:
Everyone in the class flip a coin.
Enter 1 for Heads, 2 for Tails.

Some probabilities come from combinatorial formulas. These are called **theoretic probabilities**.

Example:
An experiment involves flipping a coin.
The event $E = \{\text{Heads}\}$
Find $P(E)$.

Example:

Flip 3 fair coins. What is the probability of getting exactly two heads?

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Flip 3 fair coins. What is the probability of getting exactly two heads?

Sample space =
{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}
Event E = {HHT, HTH, THH}

P(E) = 3/8

Example:

Flip 3 fair coins. What is the probability of getting exactly two heads?

Let's do this empirically – as in do this experiment ourselves.

Click in the number of heads you get when flipping a coin 3 times.

We can use counting methods to determine both the size of the sample space, and the size of the event space to compute P(E).

CALCULATING PROBABILITY WHEN OUTCOMES ARE EQUALLY LIKELY If E is an event in a sample space S with all *equally likely outcomes*, then the probability of E is given by the formula:

$$P(E) = \frac{n(E)}{n(S)}.$$

2 roomates are selected from 2 male and 3 female applicants. If chosen at random, what is the probability that both are female?

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Sample space = 2 chosen from 5 people

Event = 2 of the 3 females chosen

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Sample space = 2 chosen from 5 people

Size = $C(5,2) = 10$

Event = 2 of the 3 females chosen

Size = $C(3,2) = 3$

$P(E) = 3/10$

Roll a die 2 times. What is the probability that the sum is 4?

Roll a die 2 times. What is the probability that the sum is 4?

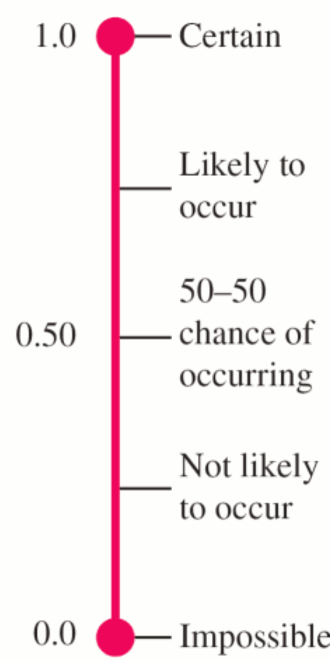
Sample space = roll a die twice

Event E = { (1,3), (2,2), (3,1) }

Counting and Probability

BASIC PROPERTIES OF PROBABILITY Assume that S is a sample space for some experiment and E is an event in S.

1. $0 \leq P(E) \leq 1$ 2. $P(\varnothing) = 0$ 3. $P(S) = 1$



Intuitive meaning of probability.

Probability and Genetics

Between 1856 and 1863, Gregor Mendel cultivated some 29,000 pea plants.

He discovered characteristics (that we now call genes) passed from parent to offspring.

One focus was on color. One gene, yellow seemed to be *dominant*. The other, green, seemed to be *recessive*.

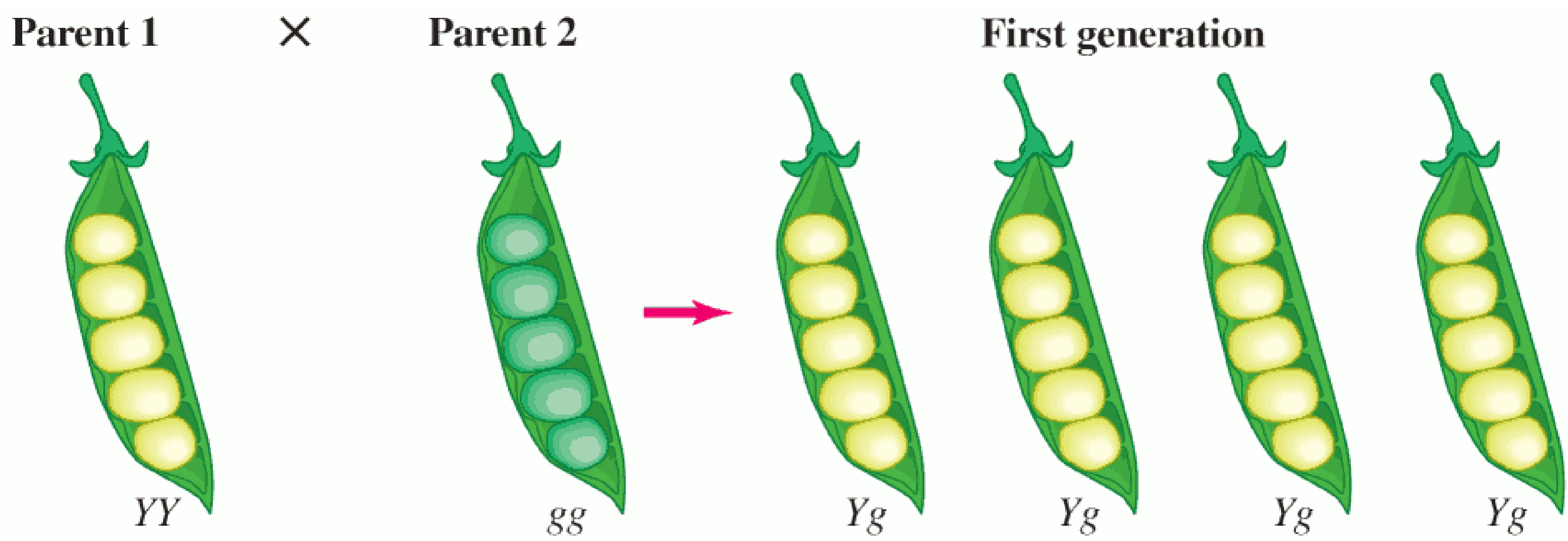
If one parent has genes AB,
and another has genes MN
then the offspring would have one of:

AM BM
AN BN

We will use uppercase for dominant genes and lowercase for recessive genes.

Probability and Genetics

Y – produces yellow seeds (dominant gene)
g – produces green seeds (recessive gene)



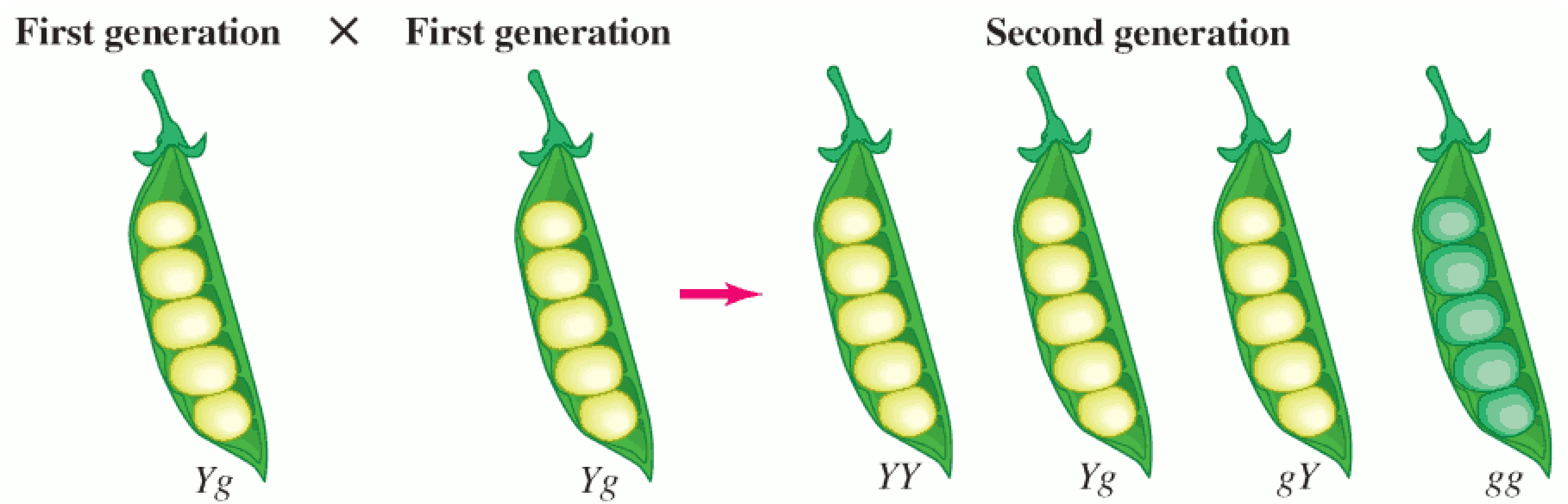
If both parents have Yg genes, what is the probability that the child of yellow pea plants is green?

Probability and Genetics

Crossing two first generation plants:

		First-Generation Plant Y	First-Generation Plant g
First-Generation Plant	Y	YY	Yg
	g	gY	gg

Punnett Square



If both parents have Yg genes, what is the probability that the child of yellow pea plants is green?

$P(E) = 1/4$

If one parent is green (gg) and the other is Yg, what is the probability that the offspring is yellow?

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Sample space = {gY, gg, gY, gg}

Event E = {gY, gY}

P(E) = 2/4 = 1/2

DEFINITION If the outcomes of a sample space are equally likely, then the **odds against an event** E are simply the number of outcomes that are against E compared with the number of outcomes in favor of E occurring. We would write these odds as $n(E'):n(E)$, where E' is the complement of event E.

The **odds for an event** is the other way around, $n(E) : n(E')$

If a family has three children, what are the odds against all three children being the same gender?

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E = same gender
= { bbb, ggg }

E' = complement = not all the same
= { bbg, bgb, bgg, gbb, gbg, ggb }

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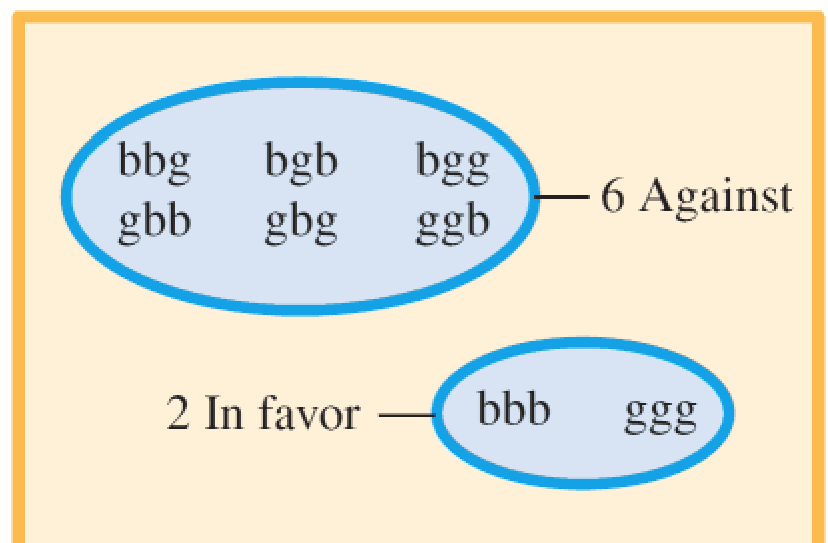
6:2 against or
3:1 against



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1:3 for

Odds

- Example: A roulette wheel has 38 equal-size compartments. Thirty-six of the compartments are numbered 1 to 36 with half of them colored red and the other half black. The remaining 2 compartments are green and numbered 0 and 00. A small ball is placed on the spinning wheel and when the wheel stops, the ball rests in one of the compartments. What are the odds against the ball landing on red?

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Odds

- Solution:

There are 38 equally likely outcomes.

Event E = “the ball lands on red”

Event E' = the opposite

Odds

- Solution:

There are 38 equally likely outcomes. 18 are in favor of the event “the ball lands on red” and 20 are against the event.

The odds against red are 20 to 18 or 20:18, which we reduce to 10:9.

Odds

PROBABILITY FORMULA FOR COMPUTING ODDS If E' is the complement of the event E, then the odds against E are

$$\frac{P(E')}{P(E)}.$$

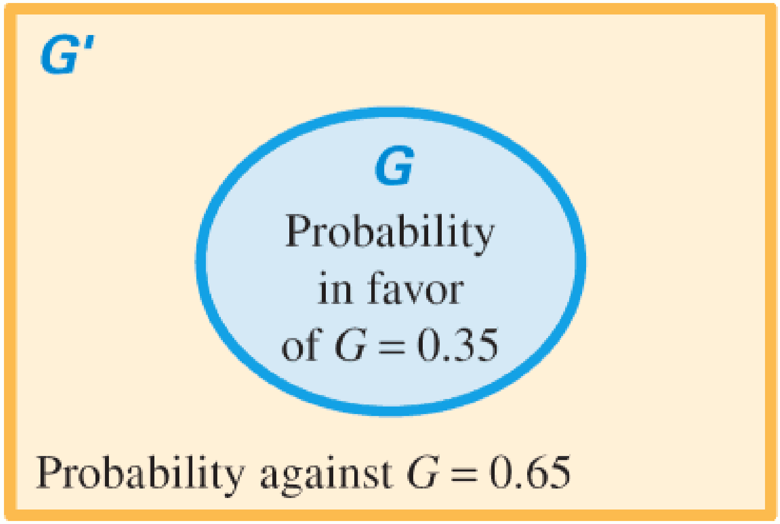
If the probability of *E* is 0.3, then the odds against *E* are

$$\text{odds against } E = \frac{\text{probability of } E'}{\text{probability of } E} = \frac{0.70}{0.30} = \frac{70}{30}.$$

We may write this as 70:30 or 7:3.

Odds

- Example: If the probability of Green Bay winning the Super Bowl is 0.35. What are the odds against Green Bay winning the Super Bowl?



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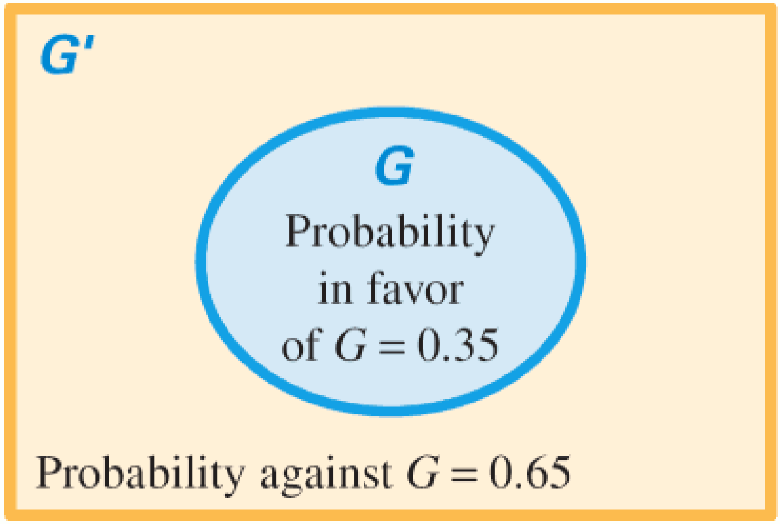

- Solution: From the diagram we compute

$$\frac{P(G')}{P(G)} = \frac{0.65}{0.35} = \frac{0.65 \times 100}{0.35 \times 100} = \frac{65}{35} = \frac{13}{7}.$$

That is, the odds against are 13 to 7.